Probability: Part 4

- Topics: Compound & Dependent Events

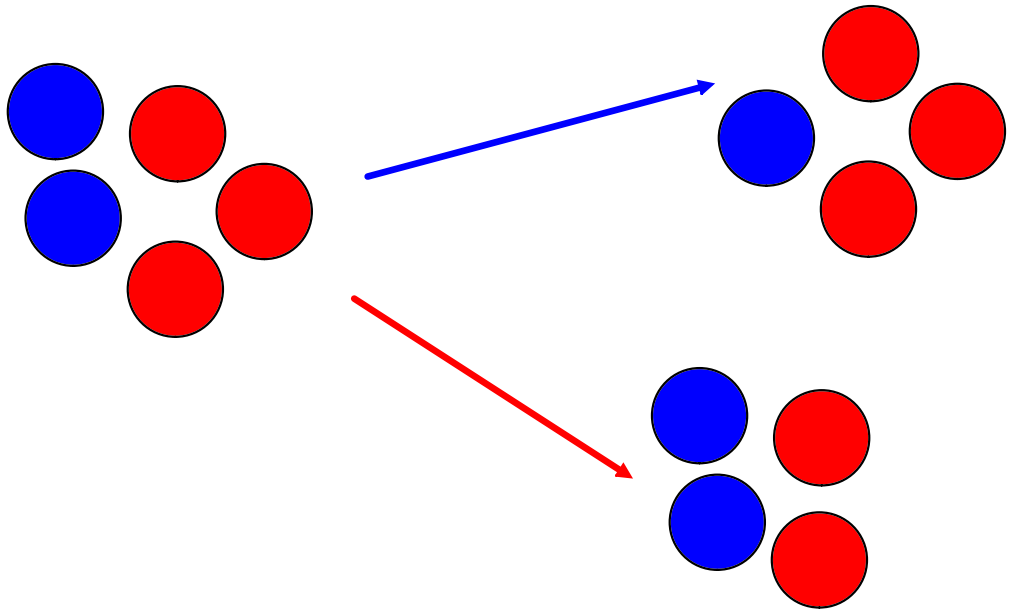
- Objective: Students will be able to calculate probabilities of dependent compound events.

- Standards: CCSS Math: 7.SP.C.8, 7.SP.C.8b, AP Stats: VAR-4 (EU), VAR-4.E (LO), VAR-4.E.1 (EK), VAR-4.E.2 (EK)

Dependent Probability

Definition: A dependent event is an event that is affected by previous events.

Example: removing colored marbles from a bag. Each time you remove a marble the chances of drawing out a certain color will change.



Dependent Probability

Example 1: Captain William has a ship, the H.M.S Crimson Lynx. The ship is five furlongs from the dread pirate Emily and her merciless band of thieves.

- If his ship hasn't already been hit, Captain William has probability 1/3 of hitting the pirate ship. If his ship has been hit, Captain William will always miss.
- If her ship hasn't already been hit, dread pirate Emily has probability 1/7 of hitting the Captain's ship. If her ship has been hit, dread pirate Emily will always miss.
- If the Captain and the pirate each shoot once, and the Captain shoots first, what is the probability that the Captain hits the pirate ship, but the pirate misses?

CH PM

$\frac{1}{3} \cdot 1 = \frac{1}{3}$

CH PH

$\frac{1}{3} \cdot 0 = 0$

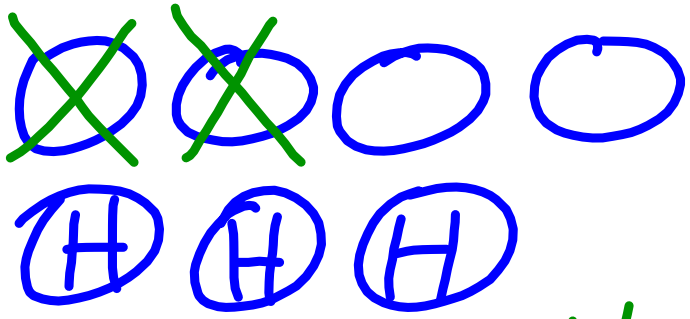
CM PH

$\frac{2}{3} \cdot \frac{1}{7} = \frac{2}{21}$

Dependent Probability

Example 2: In a class of 7, there are 3 students who have done their homework.

- If the teacher chooses 3 students, what is the probability that none of the three of them have done their homework?



$$\frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} = \frac{4}{35}$$

Conditional Probability

https://www.khanacademy.org/math/ap-statistics/probability-ap/stats-conditional-probal

Formulas you need:

- P(A and B) = P(A) * P(B|A)
- P(A|B) = P(A) and P(B|A) = P(B)

Both

10 A=Seniors

6 B=girls

P(B|A) 6/10

Given that

Conditional Probability

- P(A and B) = P(A) * P(B|A)

The nursing department of a college surveyed two hundred graduates from their programs about their current work. The college has different levels of degrees in nursing available. The department found the following probabilities:

$P(\text{bachelor's degree}) = 0.45$

$P(\text{working in nursing}) = 0.85$

$P(\text{in nursing and bachelor's}) = 0.4$

Find the probability that a graduate is currently working in nursing, given that they earned a bachelor's degree.

$P(\text{in nursing} \mid \text{bachelor's}) =$ [ ]

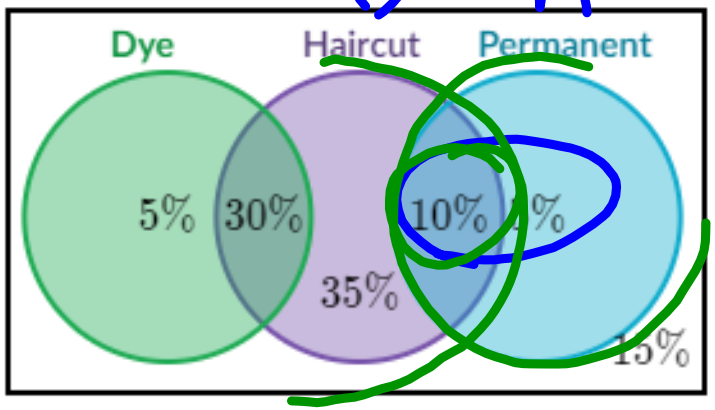
B A

$\frac{P(A \text{ and } B)}{P(A)}$ $\frac{.4}{.45} = \frac{8}{9}$

Conditional Probability

- P(A and B) = P(A) * P(B|A)

A salon owner noted what types of services its clients requested last week. Here are the results:



B A

$P(A) = .15$

$P(A \text{ and } B) = .10$

Find the probability that a randomly chosen client requested a haircut given they requested a permanent.

$P(\text{haircut} \mid \text{permanent}) =$ ____

B A

$$\frac{P(A \text{ and } B)}{P(A)} \qquad \frac{.10}{.15} = \frac{2}{3}$$

Conditional Probability

- P(A and B) = P(A) * P(B|A)

The two-way table below displays demographic data for the passengers aboard the Titanic.

	Man	Woman	Child	Total
First class	175	144	6	325
Second class	168	93	24	285
Third class	462	165	79	706
Total	805	402	109	1316

Given that a passenger selected at random was a child, find the probability that the passenger traveled in the third class.

$P(\text{third class} \mid \text{child}) =$ ____

$P(A) = \frac{109}{1316}$ $P(A \text{ and } B) = \frac{79}{1316}$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ $\frac{79}{1316}$

$\frac{P(A \text{ and } B)}{P(A)} = \frac{P(A) \cdot P(B|A)}{P(A)}$ $\frac{79/1316}{109/1316}$

$$\frac{\frac{79}{\cancel{1316}}}{\frac{109}{1316}} \cdot \frac{\cancel{1316}}{109} = \frac{79}{109}$$

Dependent and Independent Probability

- We say two events are independent if knowing one event occurred doesn't change the probability of the other event.

- Two events A and B, are independent if:
- P(A|B) = P(A) and P(B|A) = P(B)

OR

- P(A and B) = P(A) * P(B) 

Dependent and Independent Probability

- P(A|B) = P(A) and P(B|A) = P(B) or
- P(A and B) = P(A) * P(B)

Kyle and Julie are playing a game where they flip a fair coin four times and try to predict the outcomes. Using the sample space of possible outcomes listed below, answer each of the following questions.

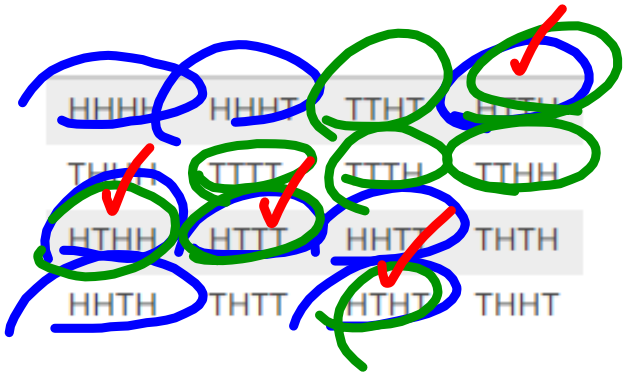
What is $P(A)$, the probability that the first flip is heads? 8/16

What is $P(B)$, the probability that the second flip is tails? 8/16

What is $P(A \text{ and } B)$, the probability that the first flip is heads *and* the second flip is tails? 4/16

Are events A and B independent? Yes

$$\frac{4}{16} = \frac{8}{16} \cdot \frac{8}{16}$$



Dependent and Independent Probability

- P(A|B) = P(A) and P(B|A) = P(B) or
- P(A and B) = P(A) * P(B)

180 students in a tenth grade high school class take a survey about which video game consoles they own. 80 students answer that one of their consoles is a Playstation, 90 answer that one of their consoles is an Xbox. Out of these, there are 30 who have both systems.

Let A be the event that a randomly selected student in the class has a Playstation and B be the event that the student has an Xbox. Based on this information, answer the following questions.

80/180

What is $P(A)$, the probability that a randomly selected student has a Playstation?

90/180

What is $P(B)$, the probability that a randomly selected student has an Xbox?

30/180

What is $P(A \text{ and } B)$, the probability that a randomly selected student has a Playstation *and* an Xbox?

30/80

What is $P(B \mid A)$, the conditional probability that a randomly selected student has an Xbox given that he or she has a Playstation?

7200

Is $P(B \mid A) = P(B)$? Are the events A and B independent?
Choose all answers that apply:

$\frac{30}{80} \neq \frac{90}{180}$

- (A) Yes, $P(B \mid A) = P(B)$.
- No, $P(B \mid A) \neq P(B)$.
- (C) Yes, events A and B are independent events.
- No, events A and B are not independent events.

Dependent and Independent Probability

Ebru has a standard deck of cards. The deck has 52 total cards and contains 4 suits: hearts, clubs, diamonds, and spades. Each suit contains cards numbered $2-10$, a jack, a queen, a king, and an ace.

Ebru randomly selects a card. Let A be the event that the card is a 2 and B be the event that it is a spade.

Which of the following statements are true?

Choose all answers that apply:

- $P(A \mid B) = P(A)$, the conditional probability that Ebru selects a 2 given that she has chosen a spade is equal to the probability that Ebru selects a 2.
- $P(B \mid A) = P(B)$, the conditional probability that Ebru selects a spade given that she has chosen a 2 is equal to the probability that Ebru selects a spade.
- Events A and B are independent events.
- The outcomes of events A and B are dependent on each other.
- $P(A \text{ and } B) = P(A) \cdot P(B)$, the probability that Ebru selects a card that is a 2 *and* a spade is equal to the probability that Ebru selects a 2 multiplied by the probability that she selects a spade.

Displaying and Comparing Quantitative Data

You should be working on the following skills:

1. Dependent probabilities
2. Calculating conditional probability
3. Dependent and independent events

Attachments

Ztable.pdf